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ROUTLEDGE

SEVENTH EDITION

BIRD'S ELECTRICAL CIRCUIT THEORY AND TECHNOLOGY

JOHN BIRD

Bird's Electrical Circuit Theory and Technology

What skills are needed for a career in electrical and electronic engineering?

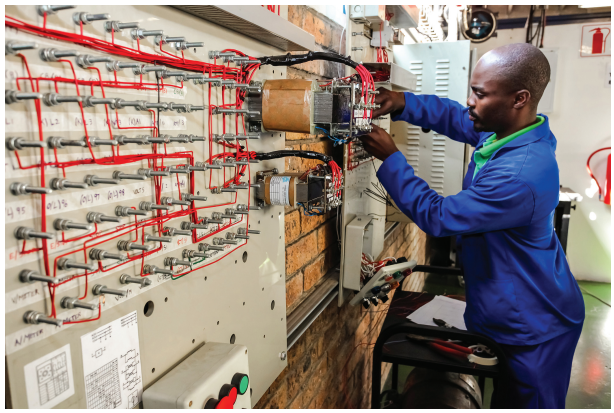
When you decide to become an electrical or electronic engineer, you're committing yourself to a profession that involves developing, designing, testing and supervising the manufacturing of electrical devices and equipment, including navigation systems, electric motors and power generation equipment. Therefore, to be able to handle such complex concepts and theories, and understand how to apply them to real-life projects, you need to possess a unique and tailored skillset. Indeed, it's no secret that a high proportion of engineering students drop out or change course, with a lack of preparedness often cited as the biggest reason for this unusually high attrition rate.
So, to see if you have what it takes to stay the course and develop a promising career in the field, here are the top 10 electrical and electronic engineering skills that you will need.

1. Problem-Solving Skills

Regardless of their discipline, engineers are, at their core, problem-solvers. This is particularly true in electrical and electronic engineering, where you are often required to think logically and apply a particular rule or concept to a problem in order to solve it.

2. Basic Circuit Knowledge

Electrical design can become an extraordinarily complex topic, especially where large installations are concerned (such as energy grids), or even within highly advanced pieces of small hardware, such as those used in smartphones. Therefore, if you are to have any hopes of getting to grips with it all, you need to first have a solid understanding of basic circuit design.



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3. Enthusiasm for Learning

Although it is an essential and unavoidable step, having a degree or a high qualification is not the end of the educational road for an electrical/electronic engineer; in fact, it is just the beginning of your active learning journey. Much of this is borne out of necessity. Electrical and electronic engineering is one of the fastest evolving and fiercely competitive engineering fields, so you will need to be constantly up to date (for example, with IEE wiring regs, and particularly if you work in the product design and manufacturing sector).

4. Communication Skills

There is barely a profession in the world where the ability to communicate is not important, and electrical and electronic engineering is no different. Whether it's understanding the needs and requirements of a client, working within project teams to develop or improve a piece of hardware/software, or working with other departments and stakeholders, communication skills are an essential part of the role.

5. Organisational Skills
The ability to organise and manage your time is important for an electrical/electronic engineer, as much of your work will likely be time-sensitive or project-based, regardless of which area of engineering you specialise in.

6. Numerical Skills
A common issue for electrical and electronic engineering students is that their mathematical background is not strong enough. Therefore, it is important to focus on mathematics at college or university. Understanding engineering is extremely difficult without a good knowledge of mathematics.

7. Work Ethic
A strong work ethic is another hugely important part of a successful engineer's makeup. Therefore, you must be determined and willing to work until you find a solution to whatever technical problems you encounter in your role.

8. Critical Thinking Skills
Critical thinking is a broad skill that can be applied to a wide array of situations, but it is just as important in electrical and electronic engineering. Possessing the ability to approach things differently or take a different view to the norm can make a big difference when you are trying to achieve a certain goal with your project.

9. Creative Thinking Skills
Engineers are not just problem-solvers - they are pioneers. Whether it's on a grand scale or a simple one, the solutions they provide change the way we live; therefore, to be able to explore and implement such radical ideas, you need to be able to think 'outside the box'. This is especially true in the commercial sector, where electronics giants are constantly competing to develop new and exciting technologies. You can have all the knowledge in the world, but if you don't know how to be creative and explore new possibilities with it, then you're going to be left behind.

10. Programming Skills
Although the importance of programming is higher in some areas of electrical and electronic engineering than others, it is still a very useful skill to possess, particularly when working with low-level embedded systems or when analysing data.

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As you can see, the career of an electrical/electronic engineer is demanding. Apart from possessing the requisite technical knowledge, it is also mandatory for you to incorporate other key soft skills into your employability repertoire, such as decision-making, leadership and attention to detail. The rewards are high though, with electrical and electronic engineering one of the highest-paying sectors in the industry.
Hopefully, Bird's Electrical Circuit Theory and Technology will help you on your first important steps in a long career in electrical and/or electronic engineering.
There is a lot to learn; stay with it - it will be worth it.

Bird's Electrical Circuit Theory and Technology

Now in its seventh edition, *Bird's Electrical Circuit Theory and Technology* explains electrical circuit theory and associated technology topics in a straightforward manner, supported by practical engineering examples and applications to ensure that readers can relate theory to practice.

The extensive and thorough coverage, containing over 800 worked examples, makes this an excellent text for a range of courses, in particular for Degree and Foundation Degree in electrical principles, circuit theory, telecommunications, and electrical technology. The text includes some essential mathematics revision, together with all the essential electrical and electronic principles for BTEC National and Diploma syllabuses and City & Guilds Technician Certificate and Diploma syllabuses in engineering. This material will be a great revision for those on higher courses.

This edition includes several new sections, including glass batteries, climate change, the future of electricity production and discussions concerning everyday aspects of electricity, such as watts and lumens, electrical safety, AC vs DC, and trending technologies.

Its companion website at **www.routledge.com/cw/bird** provides resources for both students and lecturers, including full solutions for all 1400 further questions, lists of essential formulae, and illustrations, as well as full solutions to revision tests for course instructors.

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Besides this text, ***Electrical Circuit Theory and Technology 7th Edition***, other books written by John Bird, and published by Routledge, include:

- ***Bird's Basic Engineering Mathematics 8th Edition***
- ***Bird's Engineering Mathematics 9th Edition***
- ***Bird's Higher Engineering Mathematics 9th Edition***
- ***Bird's Comprehensive Engineering Mathematics 2nd Edition***
- ***Mathematics Pocket Book for Engineers and Scientists 5th Edition***
- ***Bird's Electrical and Electronic Principles and Technology 7th Edition***
- ***Science and Mathematics for Engineering 6th Edition***
- ***Mechanical Engineering Principles 4th Edition***
- ***Mechanics of Solids 3rd Edition***

Bird's Electrical Circuit Theory and Technology

Seventh Edition

John Bird

Routledge
Taylor & Francis Group
LONDON AND NEW YORK

Seventh edition published 2022
by Routledge
2 Park Square, Milton Park, Abingdon, Oxon, OX14 4RN

and by Routledge
605 Third Avenue, New York, NY 10158

Routledge is an imprint of the Taylor & Francis Group, an informa business

First edition published by Newnes 1997
Sixth edition published by Routledge 2017

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Cataloging-in-Publication Data
Names: Bird, J. O., author
Title: Bird's electrical circuit theory and technology / John Bird.
Other titles: Electrical circuit theory and technology
Description: Seventh. | New York : Routledge, 2021. | Includes index.
Identifiers: LCCN 2021003948 (print) | LCCN 2021003949 (ebook) | ISBN 9780367672249 (hbk) | ISBN 9780367672225 (pbk) | ISBN 9781003130338 (ebk)
Subjects: LCSH: Electric circuits. | Electrical engineering.
Classification: LCC TK454 .B48 2021 (print) | LCC TK454 (ebook) | DDC 621.319/2–dc23
LC record available at https://lccn.loc.gov/2021003948
LC ebook record available at https://lccn.loc.gov/2021003949

ISBN: 978-0-367-67224-9 (hbk)
ISBN: 978-0-367-67222-5 (pbk)
ISBN: 978-1-003-13033-8 (ebk)

Typeset in Times
by KnowledgeWorks Global Ltd.

Access the companion website: www.routledge.com/cw/bird

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Preface

Bird's Electrical Circuit Theory and Technology 7th Edition provides coverage for a wide range of courses that contain electrical principles, circuit theory and technology in their syllabuses, from **Introductory to Degree level** - and including Edexcel BTEC Levels 2 to 5 National Certificate/Diploma, Higher National Certificate/Diploma and Foundation Degrees in Engineering

In this new seventh edition, **new material added** includes mention of the vast topic of global climate change and the future of electricity production, the development of glass batteries, and some practical laboratory experiments have been added at appropriate places in the text, along with other minor additions and modifications. The text is essentially, as the title suggests, all about *electrical circuit theory*, and to add too many practical descriptions would have unduly increased its extent. However, a number of associated electrical topics, hopefully of interest and help to readers, have been added, each on one or two pages, some with photographs, adding practical, everyday aspects of electricity, showing how the principles and theory explained in the text are commonly used.

The text is set out in **five sections** as follows:

SECTION 1, comprising chapters 1 and 2, involves **Revision of some basic mathematics** needed for electrical and electronic principles and in general engineering.

SECTION 2, involving chapters 3 to 14, contains '**Basic electrical engineering principles'** which any student wishing to progress in electrical engineering would need to know. An introduction to units, electrical circuits, resistance variation, batteries and alternative sources of energy, series and parallel circuits, capacitors and capacitance, magnetic circuits, electromagnetism, electromagnetic induction, electrical measuring instruments and measurements, semiconductor diodes and transistors are all included in this section.

SECTION 3, involving chapters 15 to 25, contains **'Electrical principles and technology'** suitable as a lead-in to Degree studies, and suitable for National Certificate, National Diploma and City & Guilds courses in electrical and electronic engineering. Direct current circuit theory, alternating voltages and currents, single-phase series and parallel circuits, d.c. transients, operational amplifiers, global climate change and the future of electricity production, three-phase systems, transformers, d.c. machines and three-phase induction motors are all included in this section.

SECTION 4, involving chapters 26 to 48, contains **'Advanced circuit theory and technology'** suitable for Degree, Foundation degree, Higher National Certificate/Diploma and City & Guilds courses in electrical and electronic/telecommunications engineering. The three earlier sections of the book will provide a valuable reference/revision for students at this level.

Complex numbers and their application to series and parallel networks, power in a.c. circuits, a.c. bridges, series and parallel resonance and Q-factor, network analysis involving Kirchhoff's laws, mesh and nodal analysis, the superposition theorem, Thévenin's and Norton's theorems, delta-star and star-delta transforms, maximum power transfer theorems and impedance matching, complex waveforms, Fourier series, harmonic analysis, magnetic materials, dielectrics and dielectric loss, field theory, attenuators, filter networks, magnetically coupled circuits, transmission line theory and transients and Laplace transforms are all included in this section.

SECTION 5 provides a short, **'General reference'** for standard electrical quantities - their symbols and units, the Greek alphabet, common prefixes and resistor colour coding and ohmic values.

At the beginning of each of the 48 chapters a brief explanation as to why it is important to understand the material contained within that chapter is included, together with a list of **learning objectives**.

At the end of each of the first four sections of the text is a handy reference of the **main formulae** used.

There are a number of internet downloads freely available to both students and lecturers/instructors at **www.routledge.com/cw/bird**; these are listed in the right-hand column on this page.

It is not possible to acquire a thorough understanding of electrical principles, circuit theory and technology without working through a large number of numerical problems. It is for this reason that *Bird's Electrical Circuit Theory and Technology 7th Edition* contains nearly **800 detailed worked problems**, together with some **1350 further problems (with answers at the back of the book)**, arranged within **205 Practice Exercises** that appear every few pages throughout the text. Some **1150 line diagrams** further enhance the understanding of the theory.

Fourteen Revision Tests have been included, interspersed within the text every few chapters. For example, Revision Test 1 tests understanding of chapters 3 to 6, Revision Test 2 tests understanding of chapters 7 to 9, Revision Test 3 tests understanding of chapters 10 to 14 and so on. These Revision Tests do not have answers given since it is envisaged that lecturers/instructors could set the Revision Tests for students to attempt as part of their course structure. Lecturers/instructors may obtain a complimentary set of solutions of the Revision Tests in the **Instructor's Section** at **www.routledge.com/cw/bird**

'Learning by Example' is at the heart of *Bird's Electrical Circuit Theory and Technology 7th Edition*.

JOHN BIRD
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Free Web downloads

The following support material is available from **http://www.routledge.com/cw/bird**

For Students:

1. **Full solutions to all 1350 further questions in the Practice Exercises**
2. **A set of formulae for each of the four sections of the text**
3. **68 multiple choice questions for the mathematics revision of chapters 1 and 2**
4. **Information on 38 Engineers/Scientists mentioned in the text**

For Lecturers/Instructors:

1–4. **As per students 1–4 above**

5. **Full solutions and marking scheme for each of the 14 Revision Tests; also, each test may be downloaded.**
6. **All 1150 illustrations used in the text may be downloaded for use in PowerPoint presentations**

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Section 1

Revision of some basic mathematics

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Chapter 1

Some mathematics revision

***Why it is important to understand:* Some mathematics revision**

Mathematics is a vital tool for professional and chartered engineers. It is used in electrical and electronic engineering, in mechanical and manufacturing engineering, in civil and structural engineering, in naval architecture and marine engineering and in aeronautical and rocket engineering. In these various branches of engineering, it is very often much cheaper and safer to design your artefact with the aid of mathematics – rather than through guesswork. 'Guesswork' may be reasonably satisfactory if you are designing an exactly similar artefact as one that has already proven satisfactory; however, the classification societies will usually require you to provide the calculations proving that the artefact is safe and sound. Moreover, these calculations may not be readily available to you and you may have to provide fresh calculations, to prove that your artefact is 'roadworthy'. For example, if you design a tall building or a long bridge by 'guesswork', and the building or bridge do not prove to be structurally reliable, it could cost you a fortune to rectify the deficiencies. This cost may dwarf the initial estimate you made to construct these structures, and cause you to go bankrupt. Thus, without mathematics, the prospective professional or chartered engineer is very severely disadvantaged. Using a calculator, evaluating formulae, manipulating fractions, understanding and performing calculations with percentages, appreciating ratios and direct and inverse proportion, understanding and using the laws of indices, expanding equations containing brackets, solving simple equations, transposing formulae and solving simultaneous equations are all important aspects of early mathematics that need to be revised.

Knowledge of mathematics provides the basis for all engineering.

At the end of this chapter you should be able to:

- use a calculator and evaluate formulae
- manipulate fractions
- understand and perform calculations with percentages
- appreciate ratios and direct and inverse proportion
- understand and use the laws of indices
- expand equations containing brackets
- solve simple equations
- transpose formulae
- solve simultaneous equations in two unknowns

1.1 Use of calculator and evaluating formulae

In engineering, calculations often need to be performed. For simple numbers it is useful to be able to use mental arithmetic. However, when numbers are larger an electronic calculator needs to be used.

In engineering calculations it is essential to have a **scientific notation calculator** which will have all the necessary functions needed, and more. This chapter assumes you have a **CASIO fx-991ES PLUS calculator**, or similar. If you can accurately use a calculator, your confidence with engineering calculations will improve.

Check that you can use a calculator in the following Practice Exercise.

Practice Exercise 1 Use of calculator (Answers on page 881)

1. Evaluate $378.37 - 298.651 + 45.64 - 94.562$
2. Evaluate $\dfrac{17.35 \times 34.27}{41.53 \div 3.76}$ correct to 3 decimal places
3. Evaluate $\dfrac{(4.527 + 3.63)}{(452.51 \div 34.75)} + 0.468$ correct to 5 significant figures
4. Evaluate $52.34 - \dfrac{(912.5 \div 41.46)}{(24.6 - 13.652)}$ correct to 3 decimal places
5. Evaluate $\dfrac{52.14 \times 0.347 \times 11.23}{19.73 \div 3.54}$ correct to 4 significant figures
6. Evaluate 6.85^2 correct to 3 decimal places
7. Evaluate $(0.036)^2$ in engineering form
8. Evaluate 1.3^3
9. Evaluate $(0.38)^3$ correct to 4 decimal places
10. Evaluate $(0.018)^3$ in engineering form
11. Evaluate $\dfrac{1}{0.00725}$ correct to 1 decimal place
12. Evaluate $\dfrac{1}{0.065} - \dfrac{1}{2.341}$ correct to 4 significant figures
13. Evaluate 2.1^4
14. Evaluate $(0.22)^5$ correct to 5 significant figures in engineering form
15. Evaluate $(1.012)^7$ correct to 4 decimal places
16. Evaluate $1.1^3 + 2.9^4 - 4.4^2$ correct to 4 significant figures
17. Evaluate $\sqrt{34528}$ correct to 2 decimal places
18. Evaluate $\sqrt[3]{17}$ correct to 3 decimal places
19. Evaluate $\sqrt[6]{2451} - \sqrt[4]{46}$ correct to 3 decimal places

Express the answers to questions 20 to 23 in engineering form.

20. Evaluate $5 \times 10^{-3} \times 7 \times 10^{8}$
21. Evaluate $\dfrac{6 \times 10^3 \times 14 \times 10^{-4}}{2 \times 10^6}$
22. Evaluate $\dfrac{56.43 \times 10^{-3} \times 3 \times 10^4}{8.349 \times 10^3}$ correct to 3 decimal places
23. Evaluate $\dfrac{99 \times 10^5 \times 6.7 \times 10^{-3}}{36.2 \times 10^{-4}}$ correct to 4 significant figures
24. Evaluate $\dfrac{4}{5} - \dfrac{1}{3}$ as a decimal, correct to 4 decimal places
25. Evaluate $\dfrac{2}{3} - \dfrac{1}{6} + \dfrac{3}{7}$ as a fraction
26. Evaluate $2\dfrac{5}{6} + 1\dfrac{5}{8}$ as a decimal, correct to 4 significant figures
27. Evaluate $5\dfrac{6}{7} - 3\dfrac{1}{8}$ as a decimal, correct to 4 significant figures
28. Evaluate $\dfrac{3}{4} \times \dfrac{4}{5} - \dfrac{2}{3} \div \dfrac{4}{9}$ as a fraction
29. Evaluate $8\dfrac{8}{9} \div 2\dfrac{2}{3}$ as a mixed number
30. Evaluate $3\dfrac{1}{5} \times 1\dfrac{1}{3} - 1\dfrac{7}{10}$ as a decimal, correct to 3 decimal places

31. Evaluate $\dfrac{\left(4\frac{1}{5}-1\frac{2}{3}\right)}{\left(3\frac{1}{4}\times 2\frac{3}{5}\right)}-\dfrac{2}{9}$ as a decimal, correct to 3 significant figures

In questions 32 to 38, evaluate correct to 4 decimal places.

32. Evaluate $\sin 67°$
33. Evaluate $\tan 71°$
34. Evaluate $\cos 63.74°$
35. Evaluate $\tan 39.55° - \sin 52.53°$
36. Evaluate $\sin(0.437\,\text{rad})$
37. Evaluate $\tan(5.673\,\text{rad})$
38. Evaluate $\dfrac{(\sin 42.6°)\,(\tan 83.2°)}{\cos 13.8°}$

In questions 39 to 45, evaluate correct to 4 significant figures.

39. 1.59π
40. $2.7(\pi - 1)$
41. $\pi^2\left(\sqrt{13}-1\right)$
42. $8.5e^{-2.5}$
43. $3e^{(2\pi-1)}$
44. $\sqrt{\left[\dfrac{5.52\pi}{2e^{-2}\times\sqrt{26.73}}\right]}$
45. $\sqrt{\left[\dfrac{e^{\left(2-\sqrt{3}\right)}}{\pi\times\sqrt{8.57}}\right]}$

Evaluation of formulae

The statement $\mathbf{y = mx + c}$ is called a **formula** for y in terms of m, x and c.
y, m, x and c are called **symbols**.
When given values of m, x and c we can evaluate y.
There are a large number of formulae used in engineering and in this section we will insert numbers in place of symbols to evaluate engineering quantities.
Here are some practical examples. Check with your calculator that you agree with the working and answers.

Problem 1. In an electrical circuit the voltage V is given by Ohm's law, i.e. $V = IR$. Find, correct to 4 significant figures, the voltage when $I = 5.36$ A and $R = 14.76\,\Omega$

$$V = IR = I \times R = 5.36 \times 14.76$$

Hence, **voltage V = 79.11 V, correct to 4 significant figures**

Problem 2. Velocity v is given by $v = u + at$. If $u = 9.54$ m/s, $a = 3.67$ m/s^2 and $t = 7.82$ s, find v, correct to 3 significant figures.

$$\begin{aligned} v = u + at &= 9.54 + 3.67 \times 7.82 \\ &= 9.54 + 28.6994 = 38.2394 \end{aligned}$$

Hence, **velocity v = 38.2 m/s, correct to 3 significant figures**

Problem 3. The area, A, of a circle is given by $A = \pi r^2$. Determine the area correct to 2 decimal places, given radius $r = 5.23$ m.

$$A = \pi r^2 = \pi(5.23)^2 = \pi(27.3529)$$

Hence, **area, A = 85.93 m^2, correct to 2 decimal places**

Problem 4. Density $= \dfrac{\text{mass}}{\text{volume}}$. Find the density when the mass is 6.45 kg and the volume is $300 \times 10^{-6}\,\text{m}^3$.

$$\textbf{Density} = \frac{\text{mass}}{\text{volume}} = \frac{6.45\,\text{kg}}{300\times 10^{-6}\,\text{m}^3} = \mathbf{21500\,kg/m^3}$$

Problem 5. The power, P watts, dissipated in an electrical circuit is given by the formula $P = \dfrac{V^2}{R}$. Evaluate the power, correct to 4 significant figures, given that $V = 230$ V and $R = 35.63\,\Omega$

$$P = \frac{V^2}{R} = \frac{(230)^2}{35.63} = \frac{52900}{35.63} = 1484.70390\ldots$$

Press ENG and $1.48470390.. \times 10^3$ appears on the screen
Hence, **power, P = 1485 W or 1.485 kW correct to 4 significant figures.**

Problem 6. Resistance, R Ω, varies with temperature according to the formula $R = R_0(1 + \alpha t)$. Evaluate R, correct to 3 significant figures, given $R_0 = 14.59$, $\alpha = 0.0043$ and $t = 80$

$$R = R_0(1 + \alpha t) = 14.59[1 + (0.0043)(80)]$$
$$= 14.59(1 + 0.344) = 14.59(1.344)$$

Hence, **resistance, R = 19.6 Ω, correct to 3 significant figures**

Problem 7. The current, I amperes, in an a.c. circuit is given by: $I = \dfrac{V}{\sqrt{(R^2 + X^2)}}$ Evaluate the current, correct to 2 decimal places, when V = 250 V, R = 25.0 Ω and X = 18.0 Ω

$$I = \frac{V}{\sqrt{(R^2 + X^2)}} = \frac{250}{\sqrt{(25.0^2 + 18.0^2)}} = 8.11534341\ldots$$

Hence, **current, I = 8.12 A, correct to 2 decimal places**

Now try the following Practice Exercise

Practice Exercise 2 Evaluation of formulae (Answers on page 881)

1. The area A of a rectangle is given by the formula $A = l \times b$. Evaluate the area, correct to 2 decimal places, when l = 12.4 cm and b = 5.37 cm
2. The circumference C of a circle is given by the formula $C = 2\pi r$. Determine the circumference, correct to 2 decimal places, given r = 8.40 mm
3. A formula used in connection with gases is $R = \dfrac{PV}{T}$. Evaluate R when P = 1500, V = 5 and T = 200
4. The velocity of a body is given by $v = u + at$. The initial velocity u is measured when time t is 15 seconds and found to be 12 m/s. If the acceleration a is 9.81 m/s^2 calculate the final velocity v
5. Calculate the current I in an electrical circuit, correct to 3 significant figures, when $I = V/R$ amperes when the voltage V is measured and found to be 7.2 V and the resistance R is 17.7 Ω
6. Find the distance s, given that $s = \dfrac{1}{2}gt^2$. Time t = 0.032 seconds and acceleration due to gravity g = 9.81 m/s^2. Give the answer in millimetres correct to 3 significant figures.
7. The energy stored in a capacitor is given by $E = \dfrac{1}{2}CV^2$ joules. Determine the energy when capacitance $C = 5 \times 10^{-6}$ farads and voltage V = 240 V
8. Find the area A of a triangle, correct to 1 decimal place, given $A = \dfrac{1}{2}bh$, when the base length b is 23.42 m and the height h is 53.7 m
9. Resistance R_2 is given by $R_2 = R_1(1 + \alpha t)$. Find R_2, correct to 4 significant figures, when $R_1 = 220$, $\alpha = 0.00027$ and $t = 75.6$
10. Density $= \dfrac{\text{mass}}{\text{volume}}$. Find the density, correct to 4 significant figures, when the mass is 2.462 kg and the volume is 173 cm^3. Give the answer in units of kg/m^3. Note that 1 cm^3 = 10^{-6}m^3
11. Evaluate resistance R_T, correct to 4 significant figures, given $\dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$ when $R_1 = 5.5\,\Omega$, $R_2 = 7.42\,\Omega$ and $R_3 = 12.6\,\Omega$
12. The potential difference, V volts, available at battery terminals is given by $V = E - Ir$. Evaluate V when E = 5.62, I = 0.70 and R = 4.30
13. The current I amperes flowing in a number of cells is given by $I = \dfrac{nE}{R + nr}$. Evaluate the current, correct to 3 significant figures, when n = 36. E = 2.20, R = 2.80 and r = 0.50
14. Energy, E joules, is given by the formula $E = \dfrac{1}{2}LI^2$. Evaluate the energy when L = 5.5 H and I = 1.2 A
15. The current I amperes in an a.c. circuit is given by $I = \dfrac{V}{\sqrt{(R^2 + X^2)}}$. Evaluate the

current, correct to 4 significant figures, when $V = 250\,V$, $R = 11.0\,\Omega$ and $X = 16.2\,\Omega$

1.2 Fractions

An example of a fraction is $\frac{2}{3}$ where the top line, i.e. the 2, is referred to as the **numerator** and the bottom line, i.e. the 3, is referred to as the **denominator**.

A **proper fraction** is one where the numerator is smaller than the denominator, examples being $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$, and so on.

An **improper fraction** is one where the denominator is smaller than the numerator, examples being $\frac{3}{2}$, $\frac{2}{1}$, $\frac{8}{3}$, $\frac{16}{5}$, and so on.

Addition of fractions is demonstrated in the following worked problems.

Problem 8. Evaluate A, given $A = \frac{1}{2} + \frac{1}{3}$

The lowest common denominator of the two denominators 2 and 3 is 6, i.e. 6 is the lowest number that both 2 and 3 will divide into.

Then $\frac{1}{2} = \frac{3}{6}$ and $\frac{1}{3} = \frac{2}{6}$ i.e. both $\frac{1}{2}$ and $\frac{1}{3}$ have the common denominator, namely 6.

The two fractions can therefore be added as:

$$\mathbf{A = \frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \frac{5}{6}}$$

Problem 9. Evaluate A, given $A = \frac{2}{3} + \frac{3}{4}$

A common denominator can be obtained by multiplying the two denominators together, i.e. the common denominator is $3 \times 4 = 12$

The two fractions can now be made equivalent,

i.e. $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$

so that they can be easily added together, as follows:

$$A = \frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{8+9}{12} = \frac{17}{12}$$

i.e. $$\mathbf{A = \frac{2}{3} + \frac{3}{4} = 1\frac{5}{12}}$$

Problem 10. Evaluate A, given $A = \frac{1}{6} + \frac{2}{7} + \frac{3}{2}$

A suitable common denominator can be obtained by multiplying $6 \times 7 = 42$, and all three denominators divide exactly into 42.

Thus, $\frac{1}{6} = \frac{7}{42}$, $\frac{2}{7} = \frac{12}{42}$ and $\frac{3}{2} = \frac{63}{42}$

Hence, $$A = \frac{1}{6} + \frac{2}{7} + \frac{3}{2} = \frac{7}{42} + \frac{12}{42} + \frac{63}{42}$$
$$= \frac{7+12+63}{42} = \frac{82}{42} = \frac{41}{21}$$

i.e. $$\mathbf{A = \frac{1}{6} + \frac{2}{7} + \frac{3}{2} = 1\frac{20}{21}}$$

Problem 11. Determine A as a single fraction, given $A = \frac{1}{x} + \frac{2}{y}$

A common denominator can be obtained by multiplying the two denominators together, i.e. xy

Thus, $\frac{1}{x} = \frac{y}{xy}$ and $\frac{2}{y} = \frac{2x}{xy}$

Hence, $A = \frac{1}{x} + \frac{2}{y} = \frac{y}{xy} + \frac{2x}{xy}$ i.e. $\mathbf{A = \frac{y+2x}{xy}}$

Note that addition, subtraction, multiplication and division of fractions may be determined using a **calculator**. Locate the $\frac{\square}{\square}$ and $\square\frac{\square}{\square}$ functions on your calculator (the latter function is a shift function found above the $\frac{\square}{\square}$ function) and then check the following worked problems.

Problem 12. Evaluate $\frac{1}{4} + \frac{2}{3}$ using a calculator

(i) Press $\frac{\square}{\square}$ function

(ii) Type in 1

(iii) Press $\downarrow$ on the cursor key and type in 4

(iv) $\frac{1}{4}$ appears on the screen

(v) Press $\rightarrow$ on the cursor key and type in +

(vi) Press $\frac{\square}{\square}$ function

(vii) Type in 2

(viii) Press ↓ on the cursor key and type in 3

(ix) Press → on the cursor key

(x) Press = and the answer $\frac{11}{12}$ appears

(xi) Press S ⇔ D function and the fraction changes to a decimal 0.9166666….

Thus, $\frac{\mathbf{1}}{\mathbf{4}}+\frac{\mathbf{2}}{\mathbf{3}}=\frac{\mathbf{11}}{\mathbf{12}}=\mathbf{0.9167}$ as a decimal, correct to 4 decimal places.

It is also possible to deal with **mixed numbers** on the calculator.

Press Shift then the $\frac{\square}{\square}$ function and $\square\frac{\square}{\square}$ appears.

Problem 13. Evaluate $5\frac{1}{5}-3\frac{3}{4}$ using a calculator

(i) Press Shift then the $\frac{\square}{\square}$ function and $\square\frac{\square}{\square}$ appears on the screen

(ii) Type in 5 then → on the cursor key

(iii) Type in 1 and↓on the cursor key

(iv) Type in 5 and $5\frac{1}{5}$ appears on the screen

(v) Press → on the cursor key

(vi) Type in – and then press Shift then the $\frac{\square}{\square}$ function and $5\frac{1}{5}-\square\frac{\square}{\square}$ appears on the screen

(vii) Type in 3 then → on the cursor key

(viii) Type in 3 and ↓ on the cursor key

(ix) Type in 4 and $5\frac{1}{5}-3\frac{3}{4}$ appears on the screen

(x) Press = and the answer $\frac{29}{20}$ appears

(xi) Press shift and then $S \Leftrightarrow D$ function and $1\frac{9}{20}$ appears

(xii) Press S ⇔ D function and the fraction changes to a decimal 1.45

Thus, $\mathbf{5\frac{1}{5}-3\frac{3}{4}=\frac{29}{20}=1\frac{9}{20}=1.45}$ as a decimal

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Now try the following Practice Exercise

Practice Exercise 3 Fractions (Answers on page 881)

In problems 1 to 3, evaluate the given fractions

1. $\frac{1}{3}+\frac{1}{4}$
2. $\frac{1}{5}+\frac{1}{4}$
3. $\frac{1}{6}+\frac{1}{2}-\frac{1}{5}$

In problems 4 and 5, use a calculator to evaluate the given expressions

4. $\frac{1}{3}-\frac{3}{4}\times\frac{8}{21}$
5. $\frac{3}{4}\times\frac{4}{5}-\frac{2}{3}\div\frac{4}{9}$
6. Evaluate $\frac{3}{8}+\frac{5}{6}-\frac{1}{2}$ as a decimal, correct to 4 decimal places.
7. Evaluate $8\frac{8}{9}\div 2\frac{2}{3}$ as a mixed number.
8. Evaluate $3\frac{1}{5}\times 1\frac{1}{3}-1\frac{7}{10}$ as a decimal, correct to 3 decimal places.
9. Determine $\frac{2}{x}+\frac{3}{y}$ as a single fraction.

1.3 Percentages

Percentages are used to give a common standard. The use of percentages is very common in many aspects of commercial life, as well as in engineering. Interest rates, sale reductions, pay rises, exams and VAT are all examples where percentages are used.

Percentages are fractions having 100 as their denominator.

For example, the fraction $\frac{40}{100}$ is written as 40% and is read as 'forty per cent'.

The easiest way to understand percentages is to go through some worked examples.

Problem 14. Express 0.275 as a percentage

$$0.275 = 0.275 \times 100\% = \mathbf{27.5}\%$$

Problem 15. Express 17.5% as a decimal number

$$17.5\% = \frac{17.5}{100} = \mathbf{0.175}$$

Problem 16. Express $\frac{5}{8}$ as a percentage

$$\frac{5}{8} = \frac{5}{8} \times 100\% = \frac{500}{8}\% = \mathbf{62.5}\%$$

Problem 17. In two successive tests a student gains marks of 57/79 and 49/67. Is the second mark better or worse than the first?

$$57/79 = \frac{57}{79} = \frac{57}{79} \times 100\% = \frac{5700}{79}\%$$

$$= \mathbf{72.15}\% \text{ correct to 2 decimal places.}$$

$$49/67 = \frac{49}{67} = \frac{49}{67} \times 100\% = \frac{4900}{67}\%$$

$$= \mathbf{73.13}\% \text{ correct to 2 decimal places}$$

Hence, **the second test mark is marginally better than the first test.**

This question demonstrates how much easier it is to compare two fractions when they are expressed as percentages.

Problem 18. Express 75% as a fraction

$$75\% = \frac{75}{100} = \frac{\mathbf{3}}{\mathbf{4}}$$

The fraction $\frac{75}{100}$ is reduced to its simplest form by cancelling, i.e. dividing numerator and denominator by 25.

Problem 19. Express 37.5% as a fraction

$$37.5\% = \frac{37.5}{100}$$

$$= \frac{375}{1000} \text{ by multiplying numerator and denominator by 10}$$

$$= \frac{15}{40} \text{ by dividing numerator and denominator by 25}$$

$$= \frac{\mathbf{3}}{\mathbf{8}} \text{ by dividing numerator and denominator by 5}$$

Problem 20. Find 27% of £65

$$27\% \text{ of } £65 = \frac{27}{100} \times 65 = \mathbf{£17.55} \text{ by calculator}$$

Problem 21. A 160 GB iPod is advertised as costing £190 excluding VAT. If VAT is added at 20%, what will be the total cost of the iPod?

$$\text{VAT} = 20\% \text{ of } £190 = \frac{20}{100} \times 190 = £38$$

$$\text{Total cost of iPod} = £190 + £38 = \mathbf{£228}$$

A quicker method to determine the total cost is:
$1.20 \times £190 = \mathbf{£228}$

Problem 22. Express 23 cm as a percentage of 72 cm, correct to the nearest 1%

23 cm as a percentage of 72 cm

$$= \frac{23}{72} \times 100\% = 31.94444\ldots\ldots\%$$

$$= \mathbf{32}\% \text{ correct to the nearest } 1\%$$

Problem 23. A box of screws increases in price from £45 to £52. Calculate the percentage change in cost, correct to 3 significant figures.

$$\%\text{ change} = \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%$$

$$= \frac{52 - 45}{45} \times 100\% = \frac{7}{45} \times 100 = \mathbf{15.6\%}$$

$$= \textbf{percentage change in cost}$$

Problem 24. A drilling speed should be set to 400 rev/min. The nearest speed available on the machine is 412 rev/min. Calculate the percentage over-speed.